Hybrid mode analysis of a slow-wave free-electron laser with a rectangular guide loaded with two slabs of dielectric

Ling Gen-shen and Liu Yong-gui
Department of Applied Physics, Changsha Institute of Technology, Changsha, Hunan 410073, China
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The free-electron laser of a rectangular guide loaded with two slabs of dielectric is studied by fluid theory. The main mode of super-radiation in this system is identified. This mode has all the characteristics of slow-wave free-electron lasers. Because there is no current density in this mode perpendicular to the plane of the dielectric, the electron beam can propagate near the slabs, and the growth rate may be very large.

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I. INTRODUCTION

Coherent radiation of short wavelength has long been a major object of study in the field of the free-electron laser (FEL). The disadvantages of the short-wavelength radiation of a common FEL are its high energy beam requirement and low gain [1]; the Cerenkov FEL not only has low efficiency, but also requires the thickness of the dielectric $d \simeq \lambda$ [2]. The slow-wave FEL not only has the feature of using lower beam energy to produce high gain, high efficiency, and short-wavelength coherent radiation [3,4], but it also requires no specific condition. This paper uses fluid theory to derive the dispersion equation and the gain of a slow-wave FEL of a rectangular guide loaded with two slabs of dielectric. The results are that the main mode of super-radiation in this system is the LSE mode, which has all the characteristics of slow-wave FEL's. The LSE mode is a transverse electric mode to the y direction, the direction perpendicular to the dielectric slabs. Because there is no current density J_y, the electron beam can translate near the slab, and the growth rate may be very large.

II. EIGENMODE AND DISPERSION EQUATION

Consider a metallic rectangular waveguide loaded with two slabs of dielectric (cf. Fig. 1) having a vacuum region in $t \le y \le b-t$ and two slabs of dielectric of permittivity ε in regions $0 \le y \le t$ and $b-t \le y \le t$. Generally, the eigenmodes of this system are the LSE mode and LSM mode [4,5]. The LSM mode is a transverse magnetic mode to the y direction. For the LSE mode, the scalar potential equation is written by

$$\nabla^2\phi + k^2\phi = 0 , \tag{1}$$

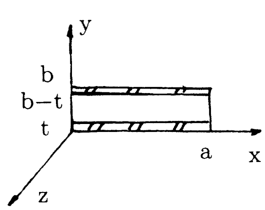

FIG. 1. Schematic of the waveguide. See text for explanation.

where $k^2=\omega^2\mu\epsilon$, k is the wave number of the microwave, ω is the frequency, and $\phi=\phi(x,y)e^{-jk_z z}$; the fields are given by

$$E_x = \frac{\partial\phi}{\partial z}, \quad H_x = \frac{1}{j\omega\mu}\frac{\partial^2\phi}{\partial x\,\partial y} ,$$
$$E_y = 0, \quad H_y = \frac{1}{j\omega\mu}\left[\frac{\partial^2\phi}{\partial y^2} + k^2\phi\right] , \tag{2}$$
$$E_z = -\frac{\partial\phi}{\partial x}, \quad H_z = \frac{1}{j\omega\mu}\frac{\partial^2\phi}{\partial y\,\partial z} .$$

Solving (1) by using the boundary conditions and continuity conditions of the fields at $y=t$ and $y=b-t$, we obtain

$$\phi = \begin{cases} A\cos\frac{m\pi}{a}x\,\sin k_1 y, & 0 \le y \le t \\ B\sin\frac{m\pi}{a}x\,(e^{-k_2 y} + e^{-k_2(b-y)}), & t \le y \le b-t \\ A\cos\frac{m\pi}{a}x\,\sin k_1(b-y), & b-t \le y \le t, \end{cases} \tag{3}$$

where k_1, k_2 are the eigenvalues of y direction in vacuum and dielectric, respectively, and they satisfy the eigen-equations

$$\frac{k_2}{k_1}\tan k_1 t = \frac{1+e^{k_2(b-2t)}}{1-e^{k_2(b-2t)}} ,$$
$$\left[\frac{m\pi}{a}\right]^2 + k_1^2 + k_z^2 = \varepsilon k_0^2 , \tag{4}$$
$$\left[\frac{m\pi}{a}\right]^2 - k_2^2 + k_z^2 = k_0^2 ,$$

where $m=1,2,3,\ldots$, and

$$\frac{A^2}{B^2} = \frac{k_2^2}{k_1^2}\frac{e^{-2k_2 t} - e^{-2k_2(b-t)}}{\cos^2 k_1 t} . \tag{5}$$

So, the fields of the LSE mode in vacuum can be written by

$$E_x=-jk_zB\sin\frac{m\pi}{a}x(e^{-k_2y}+e^{-k_2(b-y)})e^{-j(k_zz-\omega t)} ,$$

$$E_z=-\frac{m\pi}{a}B\cos\frac{m\pi}{a}x(e^{-k_2y}+e^{-k_2(b-y)})e^{-j(k_zz-\omega t)} . \tag{6}$$

There is a linear wiggler magnetic field applied to the waveguide,

$$\mathbf{B}_w=B_we^{jk_wz}\mathbf{e}_y , \tag{7}$$

where k_w is the wave number of the wiggler. An electron beam of the density

$$n_0(x,y)=n_0\Delta x_0\Delta y_0\delta\left[x-\frac{a}{2}\right]\delta\left[y-\frac{b}{2}\right] \tag{8}$$

is injected in the waveguide with a speed of $v_0\mathbf{e}_z$. Using the same method used by Tripathi and Liu in Ref. [3], we obtain the nonlinear current density and particle density,

$$J_\perp=-en'v'_{0\perp}\mathbf{e}_x ,$$

$$J_z=-\frac{\gamma_0ev_0}{j\omega'}\nabla'\cdot(n'\mathbf{v}'_0) , \tag{9}$$

$$u=\frac{\gamma_0}{j\omega'}\nabla'\cdot(n'\mathbf{v}'_0) ,$$

where $\gamma_0=(1-v_0^2/c^2)^{-1/2}$, the prime denotes Lorentz-transformed quantities in the moving frame, $\omega'=\gamma_0(\omega-v_0k_z)$, and

$$n'=\frac{e}{m(\omega'-\omega_0')^2}\left[n_0'\nabla'^2\phi_p'+\frac{\partial n_0'}{\partial x}\frac{\partial\phi_p'}{\partial x}+\frac{\partial n_0'}{\partial y}\frac{\partial\phi_p'}{\partial y}\right] ,$$

$$\mathbf{v}_0'=-\frac{eE_x'}{jm\omega'}\mathbf{e}_x , \tag{10}$$

$$E_x'=\frac{\gamma_0}{c}B\left[-jk_z+\frac{j}{\omega}v_0(k_2^2+k_0^2)\right]\times\sin\frac{m\pi}{a}x(e^{-k_2y}+e^{-k_2(b-y)})e^{-j(k_z'z'-\omega't')}$$

where $\omega_0'=\gamma_0k_wv_0$, $k_0'=-\gamma_0k_w$, $n_0'=n_0/\gamma_0$, $k_z'=\gamma_0(k_z-v_0\omega/c^2)$, $k_0^2=\omega^2/c^2$, and ϕ_p' is the ponderomotive potential,

$$\phi_p'=j\frac{\gamma_0^2e^2\beta_0BB_w}{c^2m\omega_0'\omega'}\left[-k_z+\frac{v_0}{\omega}(k_2^2+k_0^2)\right]\times\sin\frac{m\pi}{a}x(e^{-k_2y}+e^{-k_2(b-y)})\times e^{-j[(k_z'-k_0')z'-(\omega'-\omega_0')t']} , \tag{11}$$

where $\beta_0=v_0/c$.

In the conditions of strong pump and high gain, the effects of the linear current can be neglected. Using Eq. (9) in the wave equations of the radiation field, we obtain

$$\begin{aligned}\frac{\partial^2E_z}{\partial x^2}&+\frac{\partial^2E_z}{\partial y^2}-k_z^2E_z+\frac{\omega^2}{c^2}\varepsilon_rE_z\\
&=j4\pi ek_zn_1+j\frac{4\pi\omega}{x^2}J_z\\
&=j4\pi ek_z'n_1'\\
&=\frac{4\pi e^5\beta_0^2BB_w^2(k_z-v_0\omega/c^2)}{c^2m^3\gamma_0^3k_w^2v_0^2(\omega-v_0k_z)^2[\omega-(k_z+k_w)v_0]^2}\left[-k_z+\frac{v_0}{\omega}(k_2^2+k_0^2)\right]\\
&\times\frac{\partial}{\partial x}\left\{n_0\left[-\left[\frac{m\pi}{a}\right]^2+k_2^2-\gamma_0^2\left[k_z+k_w-\frac{v_0}{c^2}\omega\right]^2\right]\sin\frac{m\pi}{a}x[e^{-k_2y}+e^{-k_2(b-y)}]\right.\\
&\quad+\frac{m\pi}{a}\frac{\partial n_0}{\partial x}\cos\frac{m\pi}{a}x[e^{-k_2y}+e^{-k_2(b-y)}]\\
&\quad\left.+k_2\frac{\partial n_0}{\partial y}\sin\frac{m\pi}{a}x[e^{-k_2(b-y)}-e^{-k_2y}]\right\}e^{-j(k_zz-\omega t)} .\end{aligned} \tag{12}$$

For $t\le y\le b-t$, $\varepsilon_r'=1$; for $0\le y\le t$, $t\le y\le b-t$, $\varepsilon_r'=\varepsilon$, and the right hand side of (12) is zero. Substituting the eigenmode H_z of Eq. (6) into (12), multiplying (12) by H_z^*, and integrating over x from 0 to a, and y from 0 to b, we obtain the dispersion equation

$$\begin{aligned}\frac{a}{2}\left\{\frac{A^2}{B^2}\left[t-\frac{1}{2k_1}\sin2k_1t\right]\left[\frac{\omega^2}{c^2}\varepsilon-\left[\frac{m\pi}{a}\right]^2-k_1^2-k_z^2\right]\right.\\
+\left[2(b-2t)e^{-2k_2b}+\frac{1}{2k_2}e^{-2k_2(b-t)}(e^{-2k_2b}-1)+\frac{1}{2k_2}e^{-2k_2t}(e^{2k_2b}-1)\right]\\
\left.\times\left[\frac{\omega^2}{c^2}-\left[\frac{m\pi}{a}\right]^2+k_2^2-k_z^2\right]\right\}=\frac{P}{[\omega-(k_z+k_w)v_0]^2} ,\end{aligned} \tag{13}$$

where,

$$P=\frac{2\Delta x_0\Delta y_0\omega_{pe}^2\Omega_w^2 k_z(1-\beta_0/\eta)}{c^2\gamma_0^3 k_w^2\omega^2(1-\beta_0\eta)^2}e^{-k_2 b}\times\left[3k_z^2-8\left[\frac{m\pi}{a}\right]^2-2\gamma_0^2 k_z^2(1-\beta_0/\eta)^2\right], \quad (14)$$

and $\omega_{pe}^2=4\pi e^2 n_0/m$, $\Omega_w=eB_w/mc$, and $\eta=ck_z/\omega$. The left hand side of (13) is the eigen-dispersion equation of the LSE mode, and is nearly zero to the zeroth order; $[\omega=(k_z+k_w)v_0]^2=0$ is the linear dispersion relation of the ponderomotive potential wave. Equation (13) is the coupling dispersion equation of the LSE mode and the ponderomotive potential wave.

In order to obtain the coupling growth rate and the frequency shift, expanding the left hand side of (14) at the LSE eigenmode $\omega=\omega_1$, we obtain

$$(\omega-\omega_1)[\omega-(k_z+k_w)v_0]^2=\frac{2P\varepsilon}{\alpha a\left[\frac{2\omega_1\varepsilon}{c^2}-\frac{\partial k_1^2}{\partial w}\right]}\approx\frac{Pc^2}{\alpha a\omega_1}, \quad (15)$$

where we have used the condition of the slow variation of k_1^2 with respect to ω_1 and

$$\alpha=\varepsilon\frac{A^2}{B^2}\left[1-\frac{1}{2k_1}\sin 2k_1 t\right]+2(b-2t)e^{-2k_2 b}+\frac{1}{2k_2}e^{-2k_2(b-t)}(e^{-2k_2 b}-1)+\frac{1}{2k_2}e^{-2k_2 t}(e^{2k_2 b}-1)\,. \quad (16)$$

Solving (16) around the simultaneous zeros of the left hand side, i.e., $\omega\to\omega_1+\delta\omega\to(k_z+k_w)v_0+\delta\omega$, where $|\delta\omega|\ll\omega_1$, is the complex frequency shift, we obtain

$$(\delta\omega)^3=\frac{Pc^2}{\alpha a\omega_1}\,. \quad (17)$$

The growth mode is

$$\delta\omega=\left[\frac{Pc^2}{\alpha a\omega_1}\right]^{1/3}\left[-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right], \quad (18)$$

and the growth rate is

$$\Gamma=\mathrm{Im}(\delta\omega)=\frac{\sqrt{3}}{2}\left[\frac{Pc^2}{\alpha a\omega_1}\right]^{1/3}. \quad (19)$$

The frequency shift is

$$\mathrm{Re}(\delta\omega)=-\frac{1}{2}\left[\frac{Pc^2}{\alpha a\omega_1}\right]^{1/3} \quad (20)$$

and the efficiency is

$$\eta=\gamma_0^2\left[\frac{Pc^2}{\alpha a\omega_1^4}\right]^{1/3}. \quad (21)$$

III. DISCUSSION AND CONCLUSIONS

(i) This system is the slow-wave FEL; it has all the characteristics of slow-wave FEL's [3,4].

(ii) Because the nonlinear current density has only two components, J_x, and J_z (no J_y), the eigenmode of this system is the LSE mode. Due to the absence of J_y, the electron beam can propagate near the dielectric slabs, the interaction between the electron beam and microwave field may be very strong, and this will result in some potential benefits such as enhanced radiation growth rate, efficiency, and output power.

(iii) It can be seen from Eq. (18), that a higher growth rate may be obtained by choosing a small value a. This differs from the result of Ref. [6] where a is very large, making the $\cos(m\pi x/a)$ and $\sin(m\pi x/a)$ of Eq. (3) constant. From Eqs. (3) or (6) we can see that the larger the value of a, the smaller the eigenmode field and the interaction between the beam and the field, so the growth rate may be very small. From Eq. (19), we can also see that choosing proper parameters such as t, b, ε, B_w, and k_w may also obtain a higher growth rate.

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